

Euler's Theorems and Fleury's Algorithm

Lecture 28
Section 5.3

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Hampden-Sydney College

Fri, Mar 30, 2018

- 1 Definitions
- 2 Euler's Theorems
- 3 Fleury's Algorithm
- 4 The Splicing Algorithm
- 5 The Mail Carrier Problem Solved
- 6 Assignment

Outline

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Euler Paths and Circuits

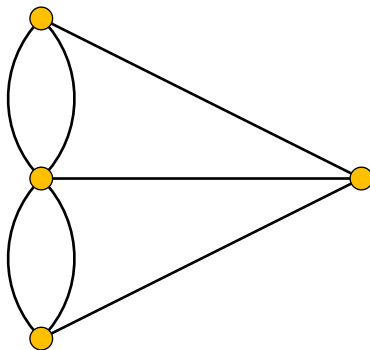
Definition (Euler Path)

An **Euler path** (pronounced "oiler") is a path that traverses each edge exactly once.

Definition (Euler Circuit)

An **Euler circuit** is an Euler path that is a circuit.

Euler Paths and Circuits



- In the Bridges of Königsberg Problem, we seek an Euler circuit (in order to return home).

Definitions

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A graph is **connected** if, for any two vertices, there is a path from one to the other.

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The **degree** of a vertex is the number of edges that emanate from it (that are “incident” to it).

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Definition (Degree)

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Definition (Even and Odd Vertices)

A vertex is **even** if it has even degree. Otherwise, it is **odd**.

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Euler's Theorems

Theorem (Euler Circuits)

If a graph

- *is connected and*
- *every vertex is even,*

then

- *it has an Euler circuit.*

Otherwise, it does not have an Euler circuit.

Euler's Theorems

Theorem (Euler Paths)

If a graph

- *is connected and*
- *has exactly 2 odd vertices,*

then

- *it has an Euler path and*
- *any Euler path must begin at one of the odd vertices and end at the other one.*

Euler's Theorems

Theorem (Euler Paths)

If a graph

- *is not connected or*
- *has more than 2 odd vertices,*

then it does not have an Euler path.

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Fleury's Algorithm

- Begin at any vertex and follow an edge.
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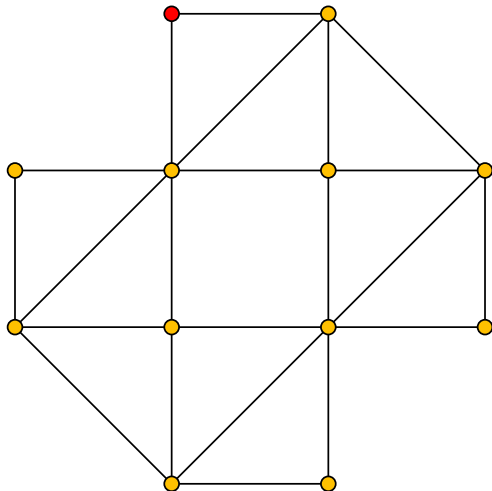
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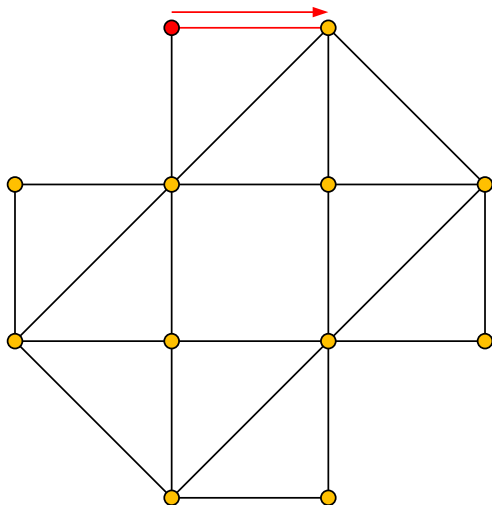
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- Continue to follow edges in this manner.
- When you reach the starting point, you have an Euler circuit.

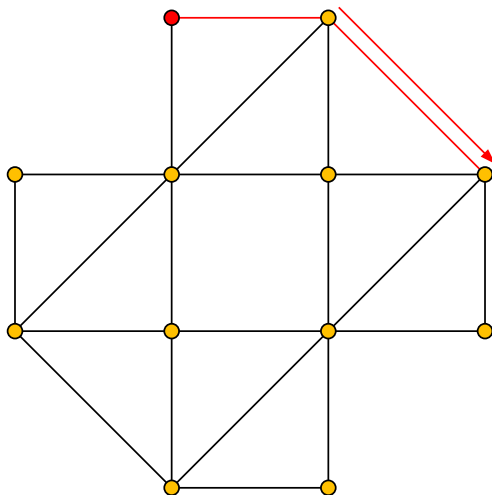
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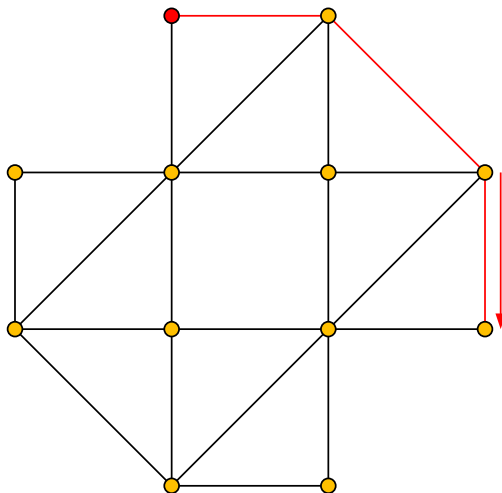
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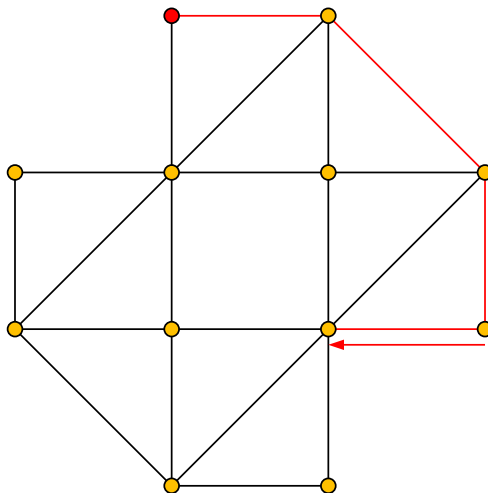
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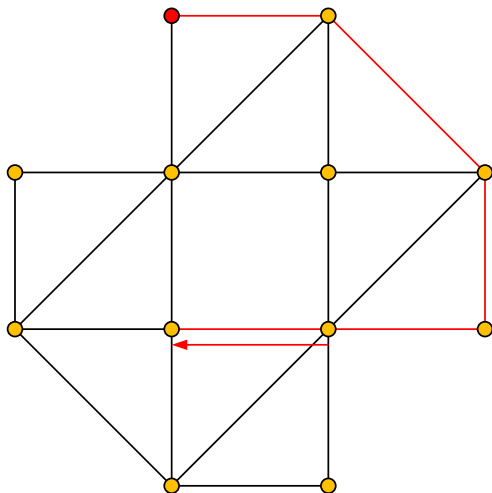
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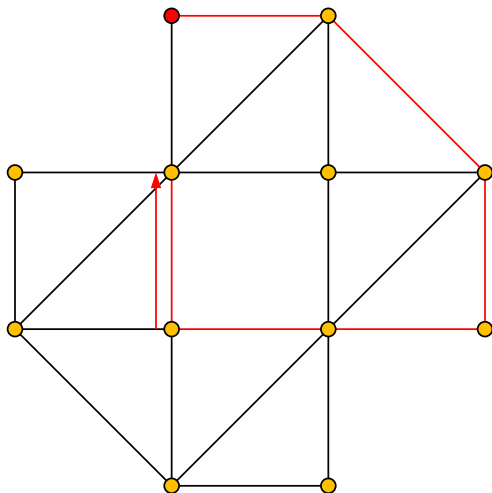
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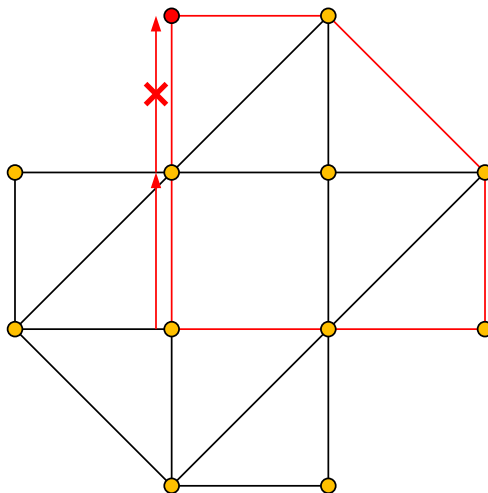
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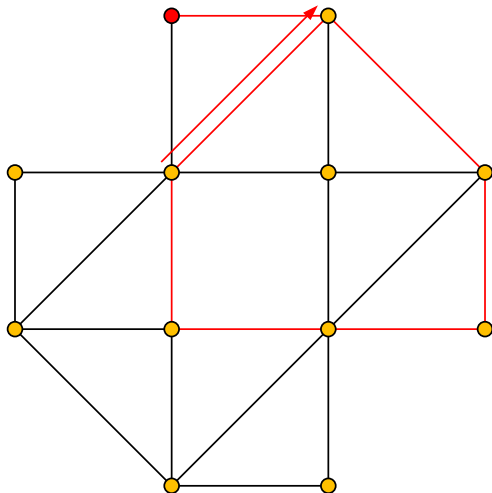
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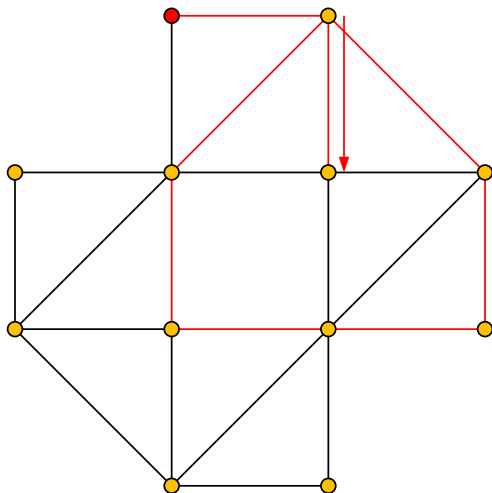
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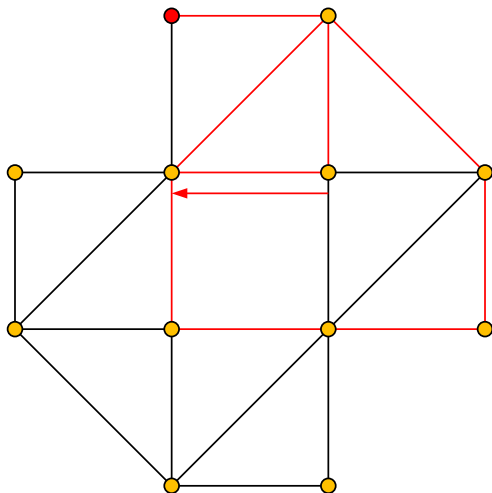
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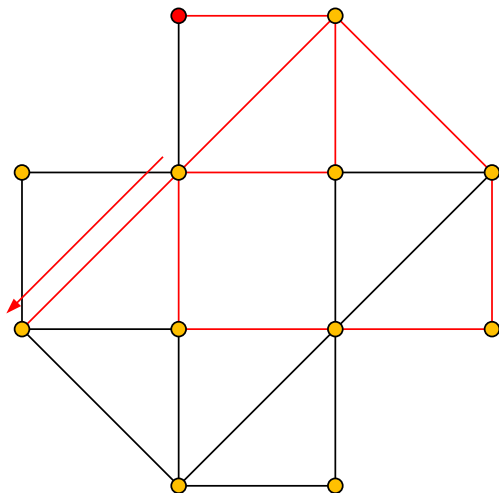
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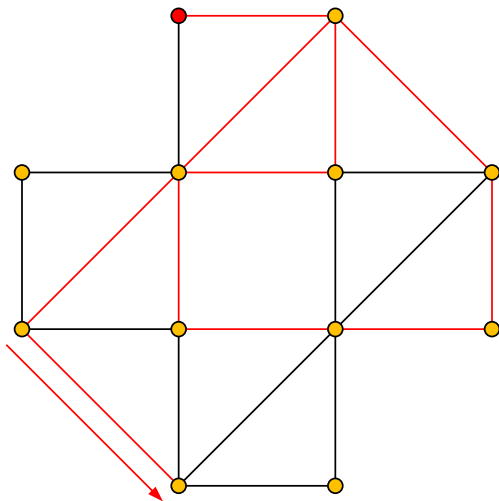
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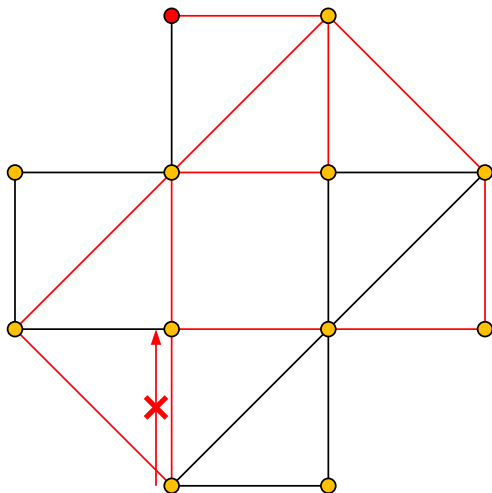
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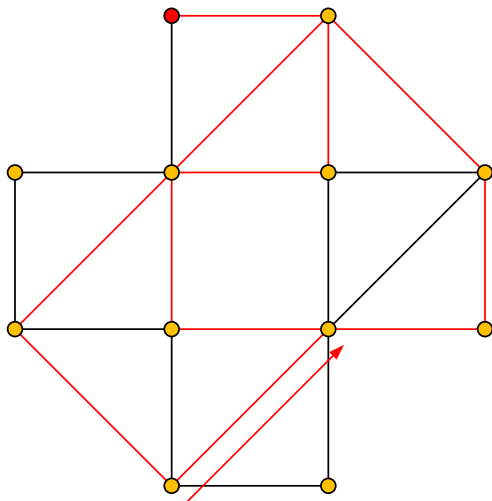
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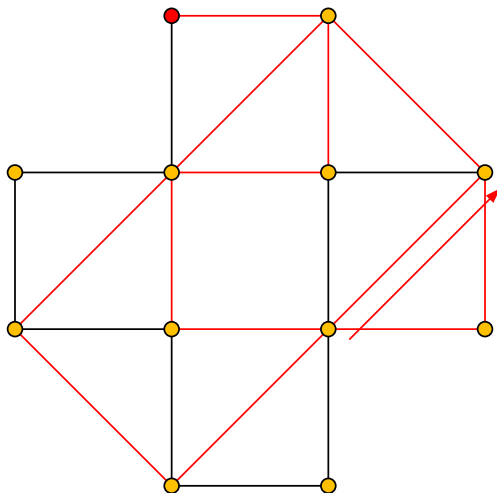
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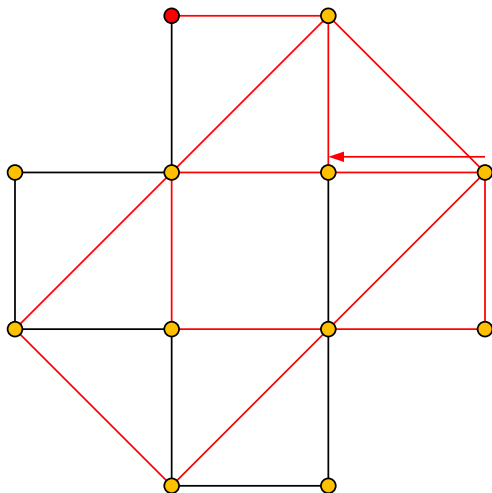
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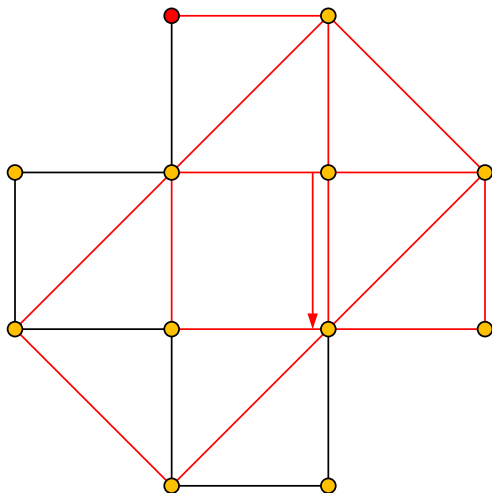
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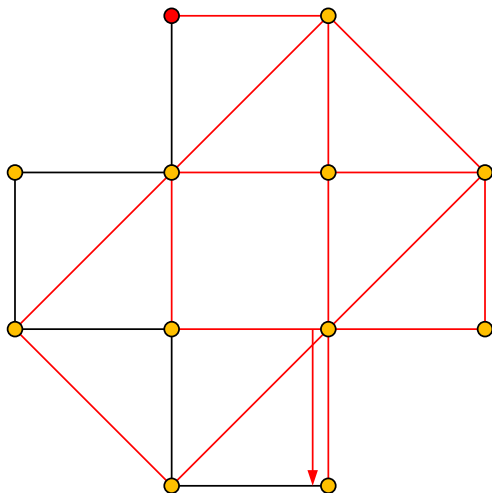
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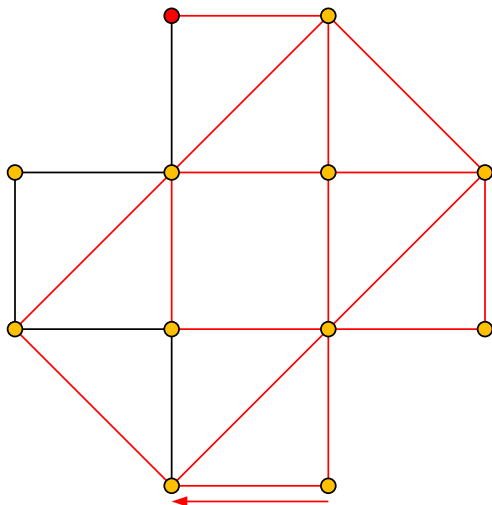
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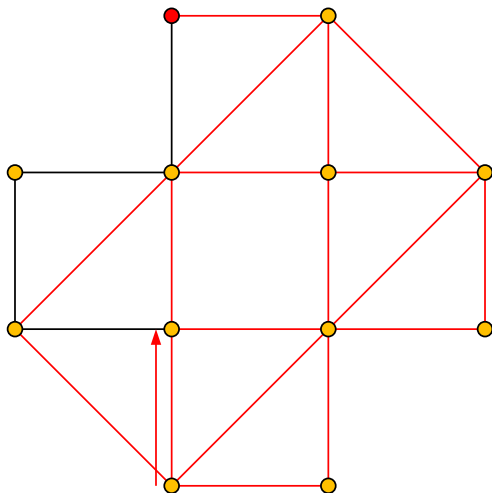
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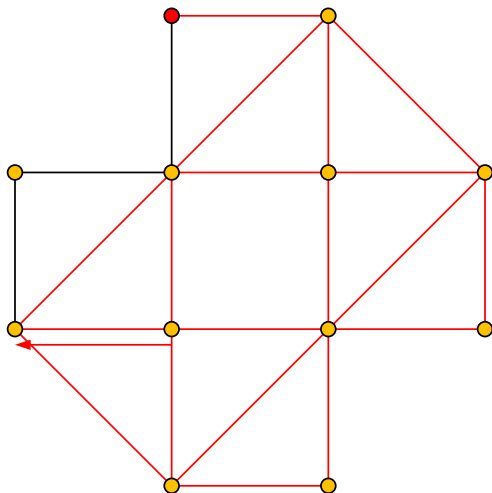
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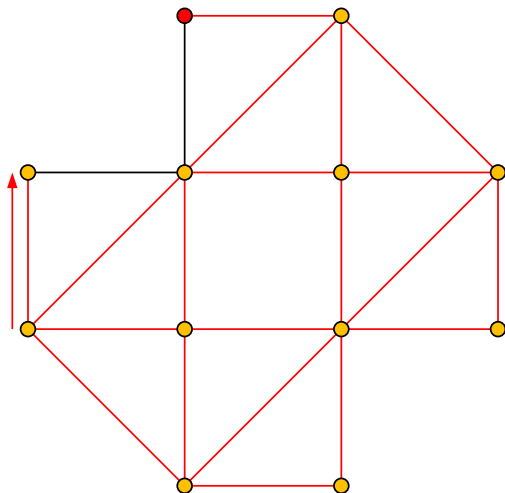
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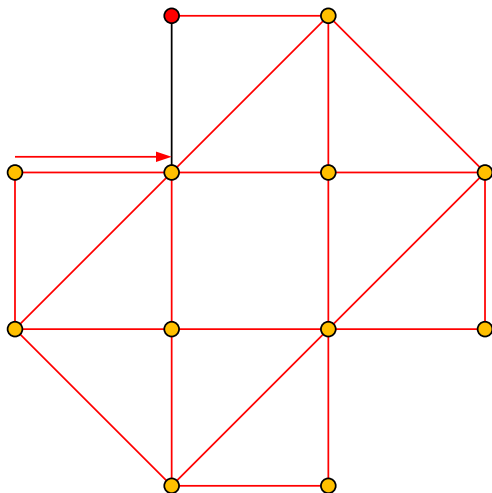
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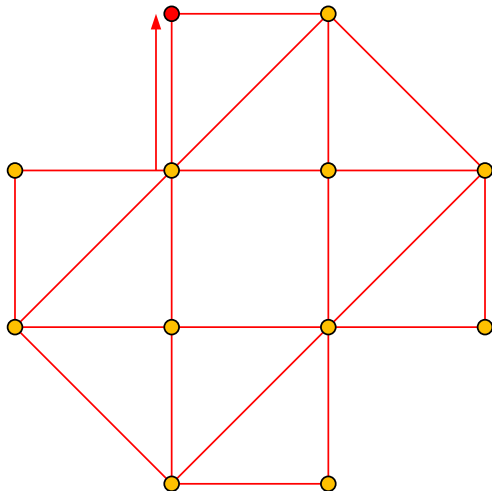
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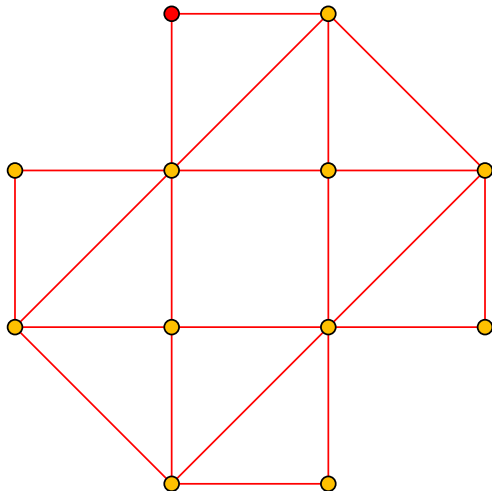
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The Splicing Algorithm

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- If there are untraveled edges, then retrace the circuit until you reach the first untraveled edge.

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- Follow untraveled edges, as before, until you return to that vertex.

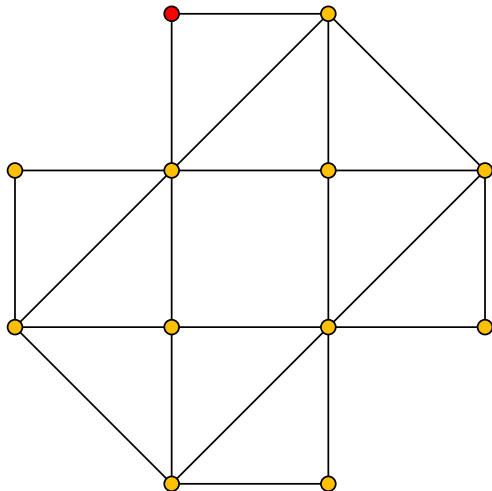
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- “Splice” that circuit into the original circuit.

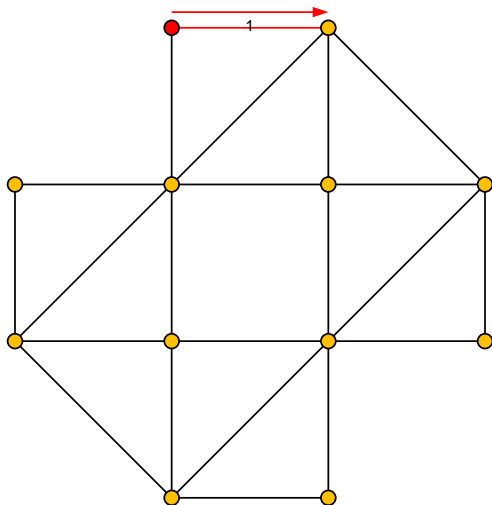
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- Follow untraveled edges, as before, until you return to that vertex.
- “Splice” that circuit into the original circuit.
- Repeat the last three steps until there are no more untraveled edges. You now have an Euler circuit.

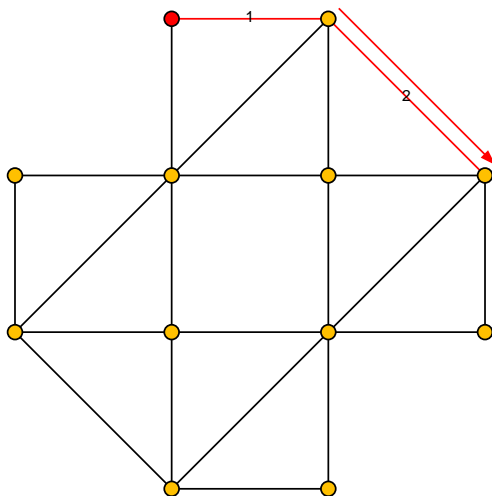
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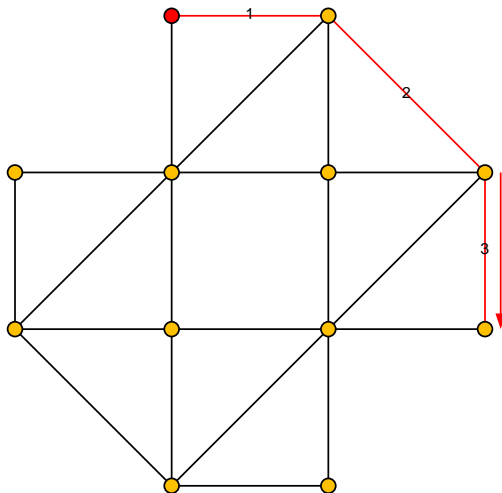
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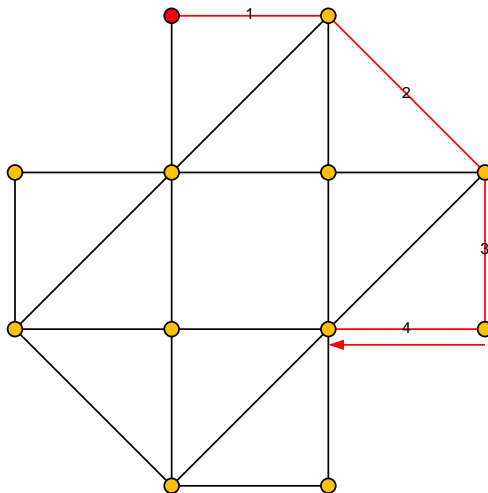
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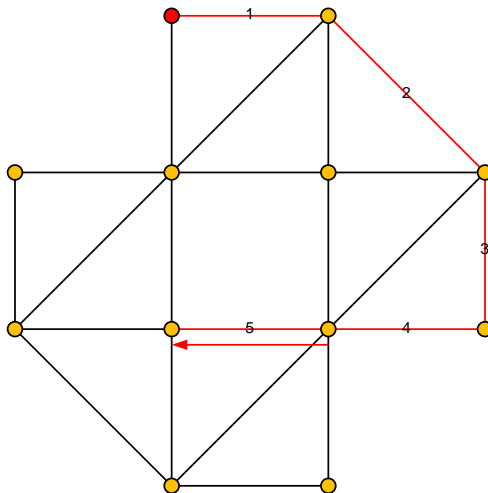
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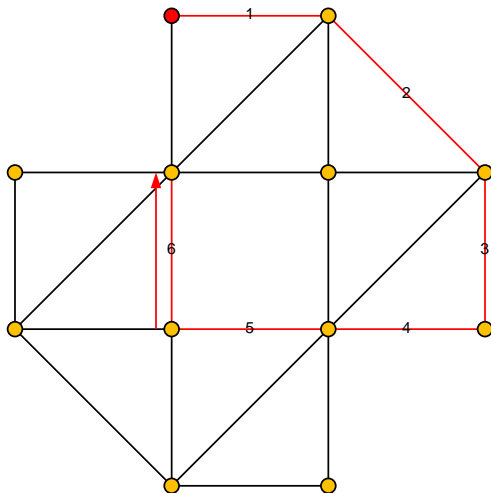
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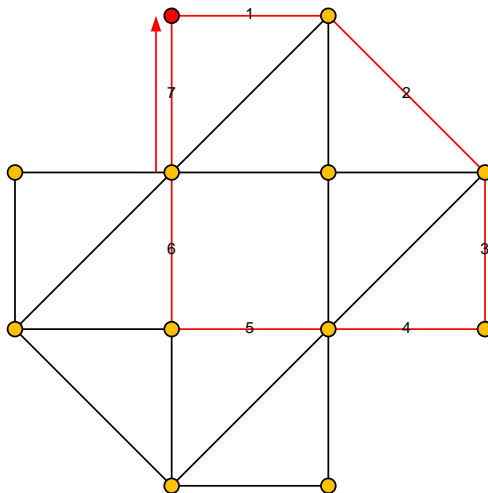
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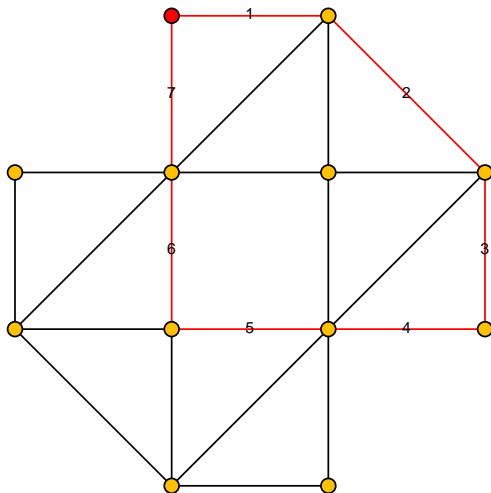
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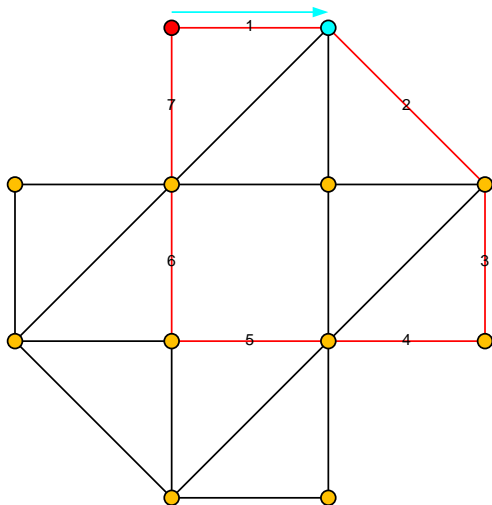
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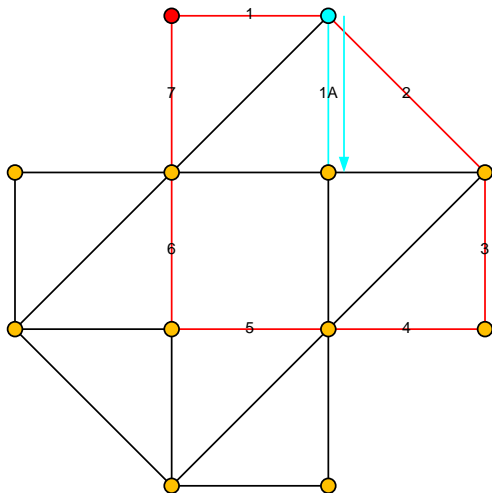
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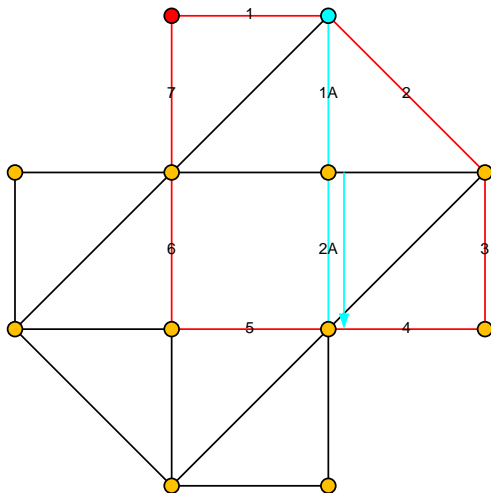
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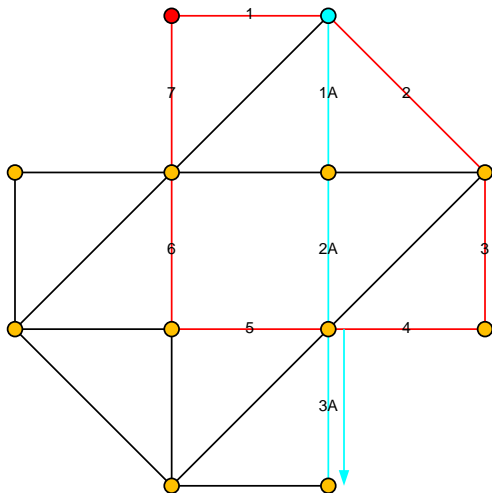
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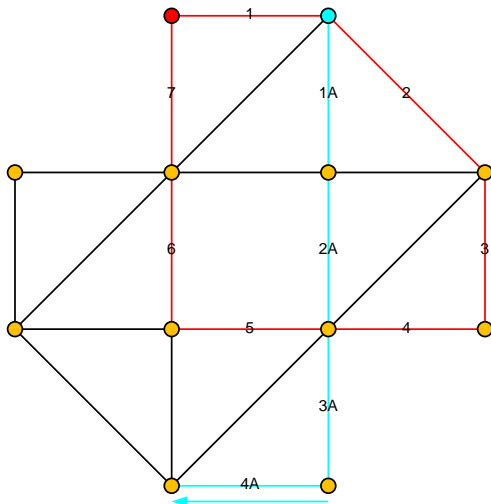
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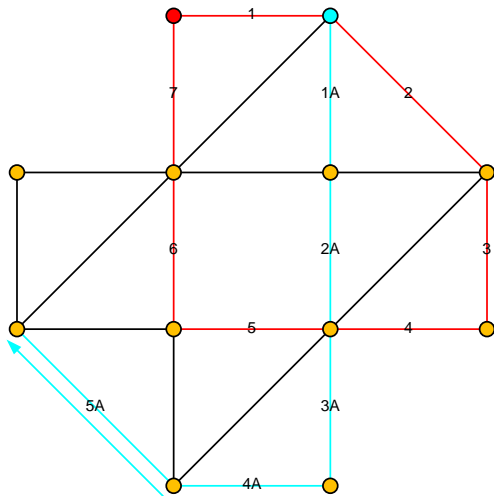
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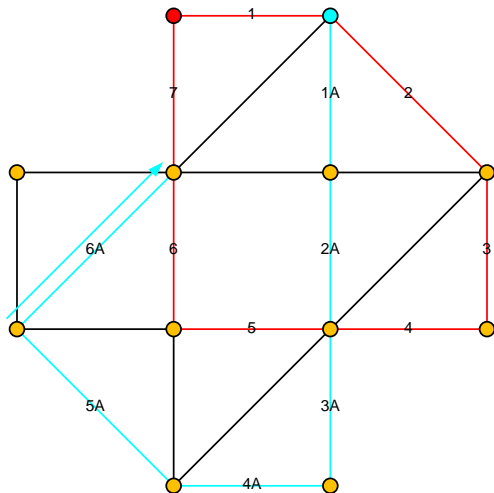
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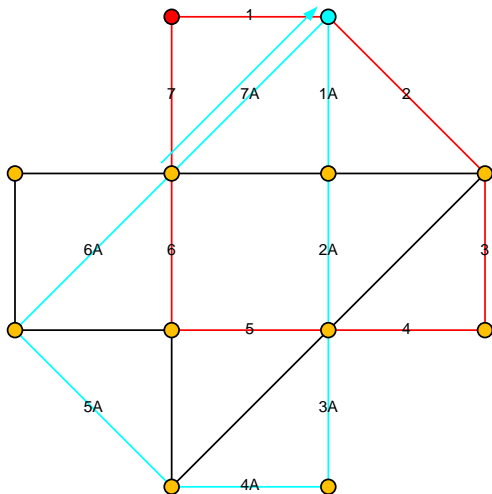
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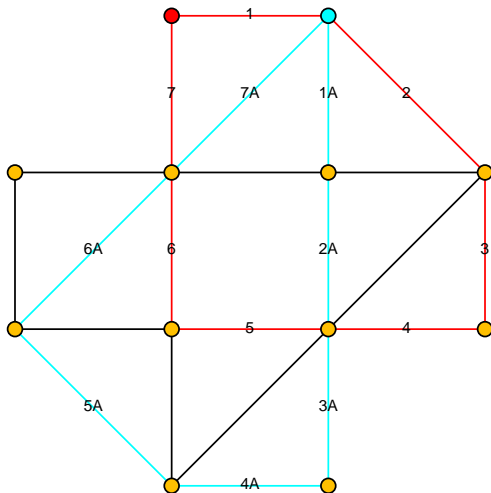
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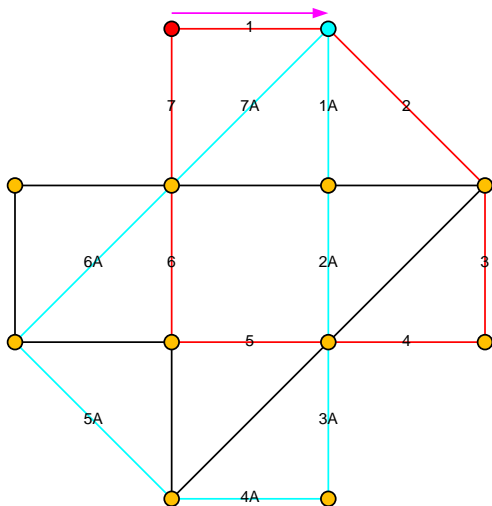
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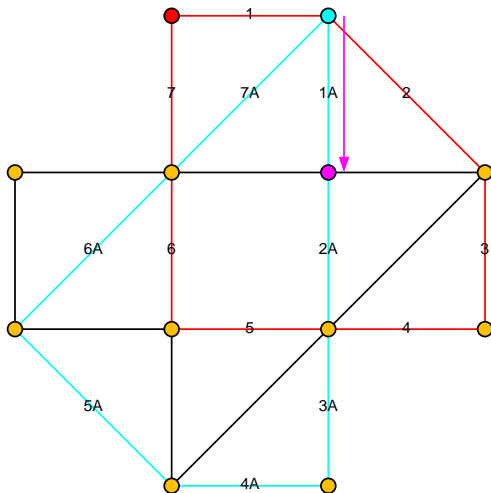
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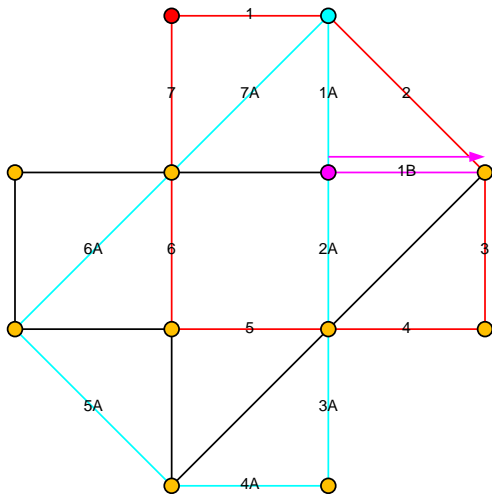
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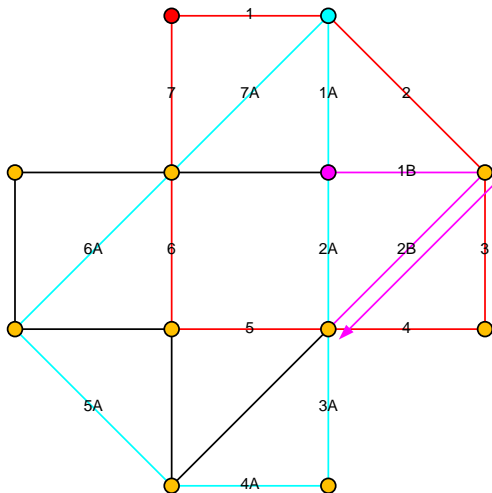
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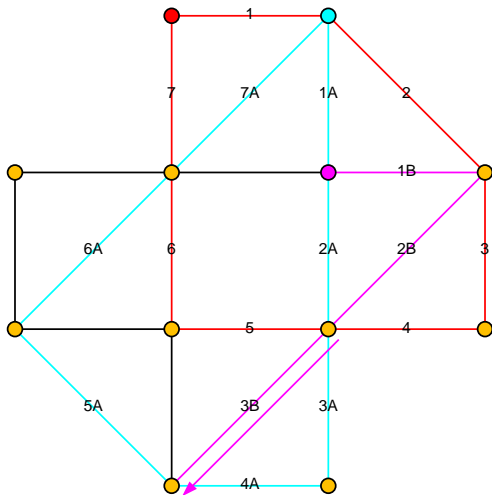
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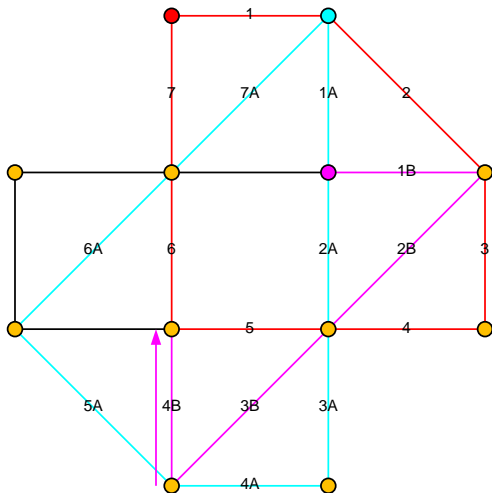
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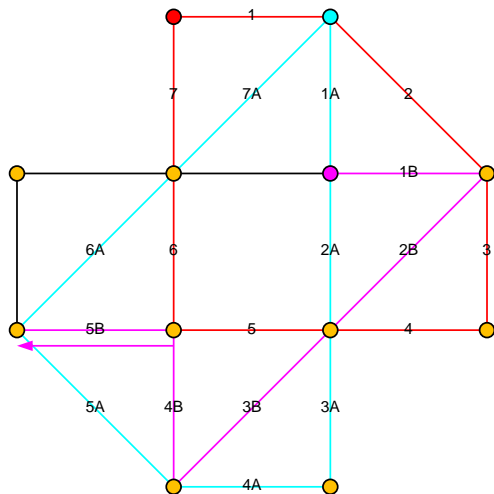
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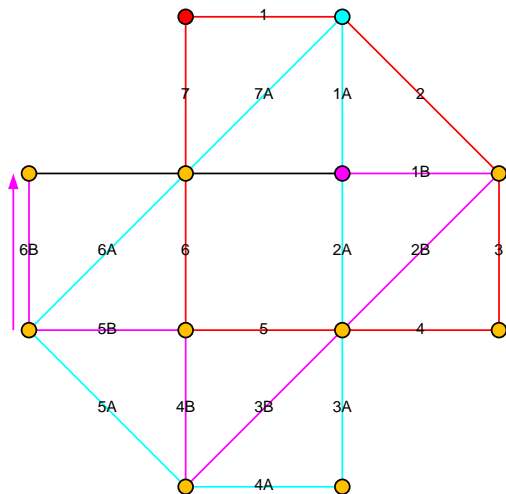
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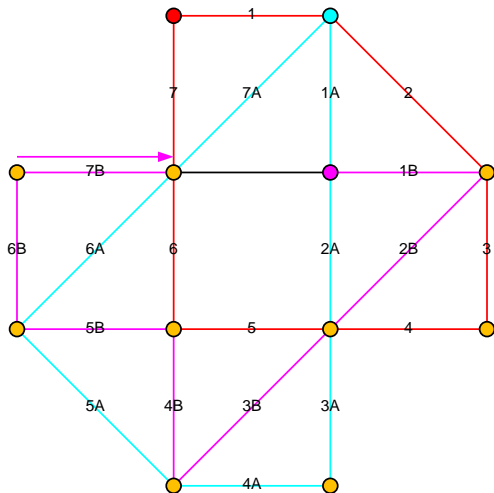
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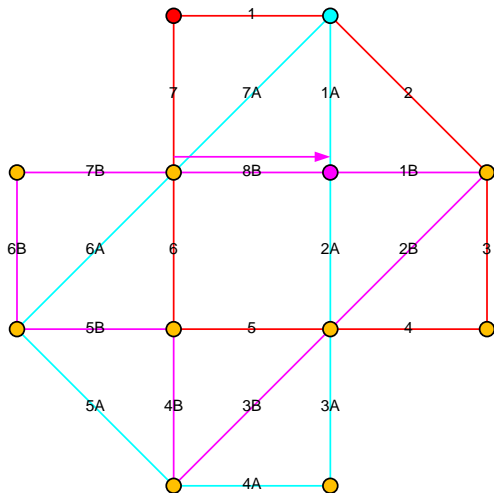
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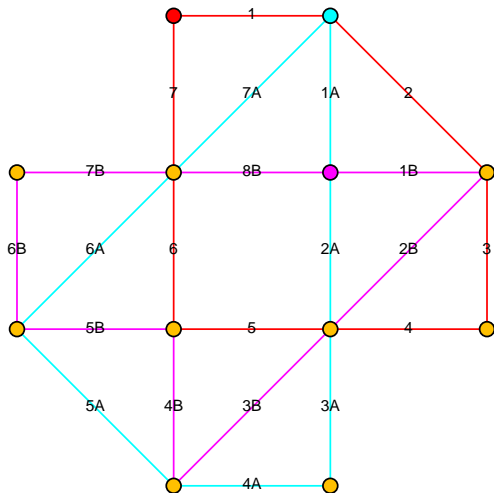
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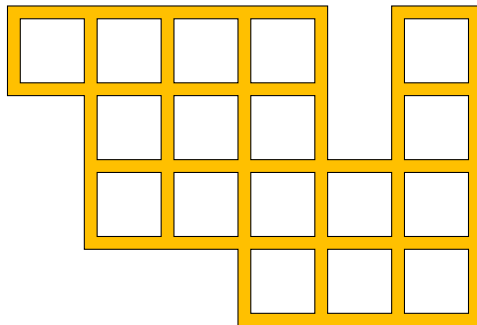
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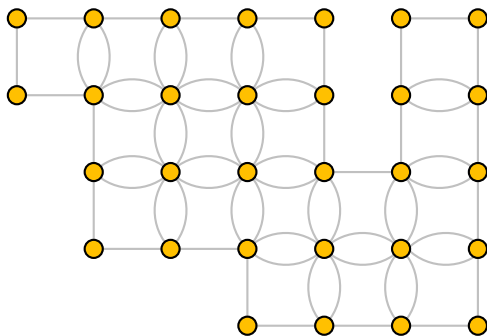
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The Mail Carrier Problem Solved



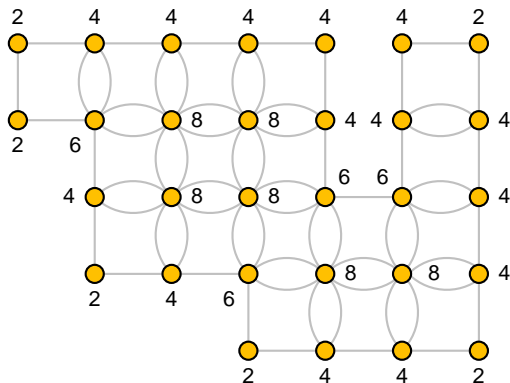
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The Mail Carrier Problem Solved



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- Chapter 5: Exercises 29, 30, 31, 35, 36, 37, 38, 39.